

FIVE-MINUTE OSCILLATION: THEORY

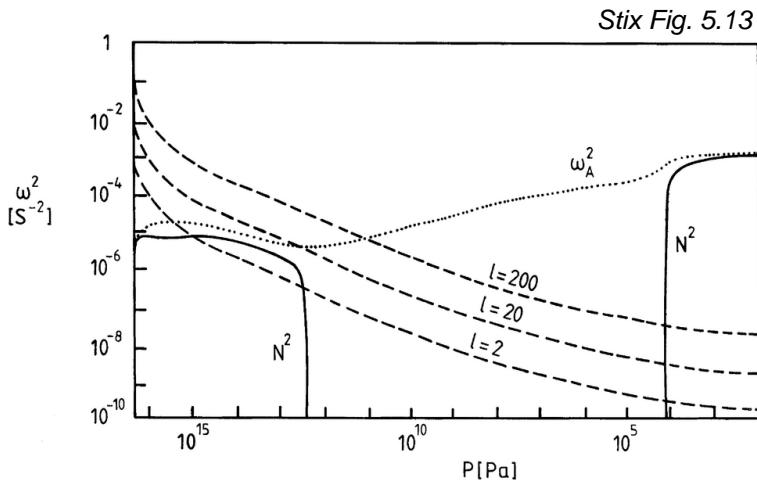
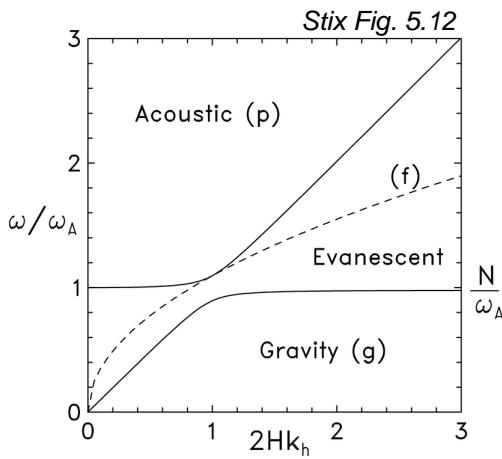
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Literature:

Jørgen Christensen-Dalsgaard: "Stellar Oscillations", 2003 [his website](#) [downloaded pdf](#)

Michael Stix: "The Sun", 2004, second edition, Springer

Rob Rutten: "Fotosferische snelheidsvelden", 1983, handwritten lecture notes



NUMERICAL p -MODE PREDICTION

Ando & Osaki 1975 PASJ...27..581A

(a) Basic Equations and Boundary Conditions

The basic equations governing nonadiabatic radial pulsations in the radiative atmosphere were formulated by UNNO (1965), in which the radiative transfer was treated in the Eddington approximation. UNNO and SPIEGEL (1966) have demonstrated that the Eddington approximation in the radiative heat equation is very useful in three-dimensional time-dependent problems. We thus utilize this in the present paper. The basic equations governing nonadiabatic nonradial oscillations in the radiative atmosphere are then given as

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho v) = 0 , \quad (1)$$

$$\frac{d v}{dt} = - \frac{1}{\rho} \nabla P - g , \quad (2)$$

$$c_P \rho \left(\frac{dT}{dt} - \nabla_{\text{ad}} \frac{T}{P} \frac{dP}{dt} \right) = -\pi \nabla \cdot F , \quad (3)$$

$$F = - \frac{4}{3\kappa\rho} \nabla J , \quad (4)$$

and

$$J = \frac{ac}{4\pi} T^4 + \frac{c_P}{4\pi\kappa} \left(\frac{dT}{dt} - \nabla_{\text{ad}} \frac{T}{P} \frac{dP}{dt} \right) , \quad (5)$$

NUMERICAL p -MODE PREDICTION

Ando & Osaki 1975 PASJ...27..581A

We assume as usual that the small perturbation of a physical quantity f is written as

$$\left. \begin{array}{c} f'(r, \theta, \varphi, t) \\ \delta f(r, \theta, \varphi, t) \end{array} \right\} = \left. \begin{array}{c} f'(r) \\ \delta f(r) \end{array} \right\} Y_l^m(\theta, \varphi) e^{i\omega t}, \quad (6)$$

where (r, θ, φ) is the spherical polar coordinates, the Eulerian and the Lagrangian perturbations are denoted by prime ('') and δ , and $Y_l^m(\theta, \varphi)$ is the spherical harmonics. We introduce a nondimensional frequency ω and five nondimensional variables x, p, θ, j , and λ defined by

$$\left. \begin{array}{l} \omega^2 = (R^3/GM)\sigma^2, \\ x = \delta r/r, \quad p = P'/\rho gr, \quad \theta = \delta T/T, \\ j = \delta J/J, \quad \lambda = \delta L_r/L_s, \end{array} \right\} \quad (7)$$

where $L_r = 4\pi^2 r^2 F$ is the luminosity at radius r and L_s is the luminosity at the surface. By linearizing equations (1)-(5), we then obtain four first-order linear differential equations and one auxiliary equation:

BASIC EQUATIONS

Stix sections 2.3.5, 5.2.1

negligible viscosity, perfect conduction, neglect rotation ($\Omega_{\text{rot}} \approx 27 \text{ days} \approx 10^4 \omega_{\text{osc}}$)

linear perturbations of hydrostatic equilibrium

subsonic velocities $v \ll c_s$ (but chromospheric shocks $M \equiv v/c_s \approx 1.1 - 1.5$)

Euler “local” versus Lagrange “material” coordinates

$$[\alpha(t+\Delta t)]_{\vec{r}} = \alpha(\vec{r}, t) + \Delta t \left(\left[\frac{\partial \alpha}{\partial t} \right]_{\vec{r}} + \sum_i \frac{\partial \alpha}{\partial x_i} \frac{\partial x_i}{\partial t} \right)$$

Lagrangian property change: δ

$$\frac{d\alpha}{dt} = \left[\frac{\alpha(t + \Delta t) - \alpha(t)}{\Delta t} \right]_{\vec{r}} = \frac{\partial \alpha}{\partial t} + \vec{v} \cdot \nabla \alpha$$

first law of thermodynamics

$$\frac{dq}{dt} = \frac{dE}{dt} + P \frac{dV}{dt} \quad V = 1/\rho \quad \rho \frac{dq}{dt} = \rho \frac{dE}{dt} - \frac{P}{\rho} \frac{d\rho}{dt}$$

ideal gas

$$\delta E = c_v \delta T \quad P = (c_p - c_v) \rho T \quad P = (\gamma - 1) \rho E \quad \gamma = \frac{c_p}{c_v} \quad \text{ionization: } \Gamma_1 = \left(\frac{\partial \ln P}{\partial \ln \rho} \right)_{\text{ad}}$$

combine

$$\frac{dP}{dt} = \frac{\gamma P}{\rho} \frac{d\rho}{dt} + (\gamma - 1) \rho \frac{dq}{dt} \quad \text{adiabatic } (\delta q = 0): \quad \frac{dP}{dt} = \frac{\gamma P}{\rho} \frac{d\rho}{dt}$$

$\delta q \neq 0$: convection (mixing length), radiation (diffusion approx. \Rightarrow Eddington approx. \Rightarrow NLTE)

CONSERVATION EQUATIONS

Stix section 5.2

continuity Euler

$$\int \frac{\partial \rho}{\partial t} dV \equiv \oint -\rho \vec{v} \cdot d\vec{s} \stackrel{\text{Gauss}}{=} - \int \nabla \cdot (\rho \vec{v}) dV \quad \frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \vec{v}) = -\vec{v} \cdot \nabla \rho - \rho \nabla \cdot \vec{v}$$

continuity Lagrange

$$\frac{d}{dt}(\rho V) = 0 \quad \frac{1}{\rho V} \frac{d}{dt}(\rho V) = \frac{1}{\rho} \frac{d\rho}{dt} + \frac{1}{V} \frac{dV}{dt} = 0 \quad \frac{dV}{dt} = V \nabla \cdot \vec{v} \quad \frac{d\rho}{dt} = -\rho \nabla \cdot \vec{v}$$

momentum

$$\rho \frac{d\vec{v}}{dt} = -\nabla P + \rho \vec{g} + \dots \quad \rho \frac{\partial \vec{v}}{\partial t} + \rho \vec{v} \cdot \nabla \vec{v} = -\nabla P + \rho \vec{g} + \dots$$

$$\text{Lorentz } (\nabla \wedge \vec{B}) \wedge \vec{B} / 4\pi \quad \text{Coriolis } 2\vec{\Omega} \wedge \vec{v} \quad \text{differential rotation } (\vec{v} \cdot \nabla \vec{\Omega}) \wedge \vec{v} \quad \text{viscosity } \mu \nabla^2 \vec{v}$$

Poisson

$$\vec{g} = -\nabla \Phi \quad \nabla^2 \Phi = 4\pi G \rho \quad \Phi(\vec{r}) = -G \int_V \frac{\rho(\vec{r}')}{|\vec{r} - \vec{r}'|} dV$$

Energy (adiabatic)

$$\frac{dP}{dt} = \frac{\gamma P}{\rho} \frac{d\rho}{dt}$$

SMALL CHANGES

Stix section 5.2.1

linear perturbations

$$P = P_0 + P_1 \quad \rho = \rho_0 + \rho_1 \quad \vec{v} = \vec{v}_0 + \vec{v}_1 = \vec{v}_1 \quad P_1 \ll P_0 \quad \rho_1 \ll \rho_0 \quad \vec{v} \ll c_s$$

Lagrangian perturbations (S 5.15 with displacement $\vec{\delta r} = \xi$)

$$\delta P = P_1 + \vec{\delta r} \cdot \nabla P_0 \quad \rho = \rho_1 + \vec{\delta r} \cdot \nabla \rho_0 \quad \vec{v} = \frac{\partial \vec{\delta r}}{\partial t}$$

continuity (S 5.13)

$$\frac{\partial \rho_1}{\partial t} + \nabla \cdot (\rho_0 \vec{v}) = 0 \quad \rho_1 + \nabla \cdot (\rho_0 \vec{\delta r}) = 0$$

momentum (S 5.14)

$$\rho_0 \frac{\partial^2 \vec{\delta r}}{\partial t^2} = \rho_0 \frac{\partial \vec{v}}{\partial t} = -\nabla P_1 + \rho_0 \vec{g}_1 + \rho_1 \vec{g}_0 = -\nabla P_1 - \rho_0 \nabla \Phi_1 + \frac{\rho_1}{\rho_0} \nabla P_0$$

Cowling approximation (S 5.2.3, 5.29): waves \Rightarrow many radial sign changes \Rightarrow average out

$$\nabla^2 \Phi_1 = 4\pi G \rho_1 \quad \Phi_1 = -G \int \frac{\rho_1(r')}{|r - r'|} dr' \approx 0$$

adiabatic energy (S 5.10)

$$\frac{P_1}{P_0} = \gamma \frac{\rho_1}{\rho_0} \quad \frac{\delta P}{P_0} = \gamma \frac{\delta \rho}{\rho_0}$$

ACOUSTIC WAVES IN HOMOGENEOUS MEDIUM

momentum, continuity, energy equations without gravity or mean-state derivatives

$$\rho_0 \frac{\partial^2 \vec{r}}{\partial t^2} = -\nabla P_1 \quad \frac{\partial \rho_1}{\partial t} = -\rho_0 \nabla \cdot \vec{v}_1 \quad P_1 = \gamma \frac{\rho_0 P_0}{\rho_1} \rho_1$$

with sound speed $c_s^2 \equiv \gamma P_0 / \rho_0$ Mach $M \equiv V/c_s$ ideal gas $c_s^2 = \gamma k T / \mu \sim T / \mu$

$$\begin{aligned} \frac{\partial^2 P_1}{\partial t^2} &= c_s^2 \left(-\rho_0 \nabla \cdot \frac{\partial \vec{v}_1}{\partial t} \right) = c_s^2 \nabla^2 P_1 \\ \frac{\partial^2 \rho_1}{\partial t^2} &= -\rho_0 \nabla \cdot \frac{\partial \vec{v}_1}{\partial t} = c_s^2 \nabla^2 \rho_1 \\ \frac{\partial^2 \vec{v}_1}{\partial t^2} &= -\frac{1}{\rho_0} \nabla \cdot \left(c_s^2 \frac{\partial \rho_1}{\partial t} \right) = c_s^2 \nabla^2 \vec{v}_1 \end{aligned}$$

space-time variable separation

$$P(x, y, z, t) \equiv g(x, y, z) f(t) \Rightarrow \frac{1}{f} \frac{\partial^2 f}{\partial t^2} = \frac{c_s^2}{g} \nabla^2 g \equiv -\omega^2 \Rightarrow \frac{\partial^2 f}{\partial t^2} + \omega^2 f = 0 \Rightarrow f(t) = a_t e^{i\omega t} + b_t e^{-i\omega t}$$

spatial variable separation $g(x, y, z) = g_x(x) g_y(y) g_z(z)$

$$\nabla^2 g + \frac{\omega^2}{c_s^2} g = 0 \Rightarrow \frac{\partial^2 g_x}{\partial x^2} + k_x^2 g_x = 0 \Rightarrow g_x(x) = a_x e^{ik_x x} + b_x e^{-ik_x x} \text{ etc.}$$

plane waves

$$P_1 = P_0 \gamma M e^{i(\vec{k} \cdot \vec{r} \pm i\omega t)} \quad \rho_1 = \rho_0 M e^{i(\vec{k} \cdot \vec{r} \pm i\omega t)} \quad \vec{v}_1 = c_s M e^{i(\vec{k} \cdot \vec{r} \pm i\omega t)}$$

spatial wavenumber $\vec{k} = (k_x, k_y, k_z) \parallel \vec{v}$

dispersion relation $k^2 = |\vec{k}|^2 = \omega^2 / c_s^2$

PLANE WAVE PROPERTIES

waves in homogeneous gaseous medium

$$\frac{P_1}{P_0 \gamma M} = \frac{\rho_1}{\rho_0 M} = \frac{\vec{v}_1}{c_s M} = e^{i(\vec{k} \cdot \vec{r} - \omega t)} \quad \vec{k} \parallel \vec{v}_1 \parallel \delta \vec{r} \text{ longitudinal}$$

wavelength, wavenumber, period, (angular) frequency

$$\text{full cycle at } kx = 2\pi \Rightarrow x = \lambda \quad k_x = 2\pi/\lambda$$

$$\text{full cycle at } \omega t = 2\pi \Rightarrow t = P \quad \omega = 2\pi/P = 2\pi f$$

plane wave: same phase across plane with $\vec{k} \cdot \vec{r} = 0$ so $\perp \vec{k}$ and $\perp \vec{v}$

phase velocity = propagation wave pattern

$$\text{wave } \vec{v} = (\dot{x}, 0, 0): \text{ equal phase: } k_x x - \omega t = c \quad x = (\omega/k_x) t + c \quad v_{\text{phase}} = \omega/k = c_s$$

group velocity = propagation envelope non-monochromatic wave train $L_x \Delta k_x \approx 1/2$

$$e^{i[(\vec{k} + \Delta \vec{k}) \cdot \vec{r} - (\omega + \Delta \omega)t]} = e^{i(\vec{k} \cdot \vec{r} - \omega t)} e^{i(\Delta \vec{k} \cdot \vec{r} - \Delta \omega t)} \Rightarrow v_{\text{group}} = \frac{\partial \omega}{\partial k}$$

dispersion relation $|\vec{k}|^2 = k_x^2 + k_y^2 + k_z^2 = \omega^2/c_s^2$ and diagnostic diagram

shaded: real solutions = propagating waves

boundary line: $k_y = k_z = 0$, waves only in x

smaller k_x towards upper left

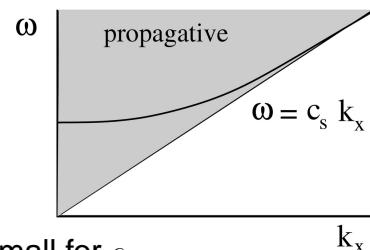
ω axis: $k_x = 0$, waves only in y and/or z

curve: $\omega = \sqrt{c_s^2 k_x^2 + c}$ for constant $k_y^2 + k_z^2 = c$

long periods: only at large wavelength $\lambda_x = 2\pi/k_x$

non-shaded: period P too long for k_x , wavelength λ_x too small for c_s

imaginary solutions $k_y^2 < 0$ or $k_z^2 < 0$: exponential growth/decay in y or z



VERTICAL ACOUSTIC WAVES IN ISOTHERMAL ATMOSPHERE

“atmosphere” \equiv gas layer plane-parallel in x and y , stratified by gravity in z

$$\nabla T_0 = 0 \quad \nabla P_0 = \rho_0 \vec{g} \Rightarrow \frac{\partial P_0}{\partial z} = -\rho_0 g = -P_0 \frac{\gamma g}{c_s^2} \equiv -\frac{P_0}{H} \quad H = \frac{c_s^2}{\gamma g} = \frac{RT}{\mu g} \quad k_h^2 \equiv k_x^2 + k_y^2$$

linearized conservation equations with $\vec{v} = (v_h, v_z)$ and $\vec{g} = (0, g)$

$$\frac{\partial \rho_1}{\partial t} = -\rho_0 \nabla \cdot \vec{v} - \vec{v} \cdot \nabla \rho_0 \quad \rho_0 \frac{\partial \vec{v}}{\partial t} = \rho_1 \vec{g} - \nabla P_1 \quad \frac{\partial P_1}{\partial t} + \vec{v} \cdot \nabla P_0 = -\gamma P_0 \nabla \cdot \vec{v}$$

give inhomogeneous wave equations for P_1, ρ_1, \vec{v}

$$\frac{\partial^2 \vec{v}}{\partial t^2} = c_s^2 \nabla^2 \vec{v} - (\gamma - 1) \vec{g} \nabla \cdot \vec{v} - \nabla(\vec{v} \cdot \vec{g})$$

vertical motion $\vec{v} = (0, v_z)$

$$\frac{\partial^2 v_z}{\partial t^2} = c_s^2 \nabla^2 v_z - \gamma g \frac{\partial v_z}{\partial z} \quad \text{try } v_z = e^{i(k_z z - \omega t)}$$

$$\omega^2 = c_s^2 k_z^2 + i \gamma g k_z \quad k_z = -i \frac{\gamma g}{2c_s^2} \pm \frac{1}{c_s} \sqrt{\omega^2 - \omega_a^2} \quad \omega_a \equiv \frac{\gamma g}{2c_s}$$

$$v_z = e^{(\gamma g / 2c_s^2) z} e^{i [\pm(z/c_s) \sqrt{\omega^2 - \omega_a^2} - \omega t]} = e^{z/2H} e^{i [\pm(z/c_s) \sqrt{\omega^2 - \omega_a^2} - \omega t]}$$

properties

amplitude $\sim e^{z/2H} \sim \sqrt{\rho_o(z)} =$ energy conservation $(1/2)\rho_0 v_z^2$ up to non-linear regime

$\omega > \omega_a$ propagating plane wave

$\omega < \omega_a$ slow perturbation = evanescent wave: whole atmosphere up and down in phase
aperiodic growth or decay with z depending on upper or lower piston

SLANTED AG WAVES IN ISOTHERMAL ATMOSPHERE 1

Stix section 5.2.4 Hines 1960 CaJPh..38.1441H Whitaker 1963 ApJ...137..914W

substitute into linearised conservation laws

$$\frac{P_1}{P_0 \mathcal{P}} = \frac{\rho_1}{\rho_0 \mathcal{R}} = \frac{v_h}{\mathcal{X}} = \frac{v_z}{\mathcal{Z}} = e^{i(k_h \cdot x + k_z \cdot z - \omega t)}$$

complex $\mathcal{P}, \mathcal{R}, \mathcal{X}, \mathcal{Z}, k_h, k_z$

separation of vertical-only solution: $\Im k_z = \gamma g / 2c_s^2$

$$\frac{P_1}{P_0 \mathcal{P}} = \frac{\rho_1}{\rho_0 \mathcal{R}} = \frac{v_h}{\mathcal{X}} = \frac{v_z}{\mathcal{Z}} = e^{z/2H} e^{i(k_h x + k_z z - \omega t)}$$

real k_h, k_z

polarisation relations

$$\begin{aligned} \mathcal{P} &= \gamma \omega^2 \left[k_z - i \left(1 - \frac{\gamma}{2}\right) \frac{g}{c_s^2} \right] & \mathcal{R} &= \omega^2 k_z + i (\gamma - 1) g k_h^2 - i \frac{\gamma}{2} \frac{g \omega^2}{c_s^2} \\ \mathcal{X} &= \omega k_h c_s^2 \left[k_z - i \left(1 - \frac{\gamma}{2}\right) \frac{g}{c_s^2} \right] & \mathcal{Z} &= \omega (\omega^2 - k_h^2 c_s^2) \end{aligned}$$

amplitude & phase

$$\mathcal{X} \equiv |\mathcal{X}| e^{i\phi} \quad v_h = |\mathcal{X}| e^{z/2H} e^{i(k_h x + k_z z - \omega t + \phi)} \quad \phi = \arctan(\Im X / \Re X)$$

dispersion relation

$$\omega^4 - \omega^2 c_s^2 (k_h^2 + k_z^2) + (\gamma - 1) g^2 k_h^2 - \frac{\gamma^2 g^2 \omega^2}{4c_s^2} = 0$$

$$\omega_a \equiv \frac{\gamma g}{2c_s} \quad \omega_g \equiv \frac{g}{c_s} \sqrt{\gamma - 1} \quad \omega_g < \omega_a \text{ since } \gamma < 2 \quad \gamma = 5/3 \Rightarrow \omega_g = 0.98 \omega_a$$

$$(\omega^2 - \omega_a^2) \frac{\omega^2}{c_s^2} - \omega^2 (k_h^2 + k_z^2) + \omega_g^2 k_h^2 = 0$$

SLANTED AG WAVES IN ISOTHERMAL ATMOSPHERE 2

reordered dispersion relations

$$k_h^2 = \frac{(\omega_a^2 - \omega^2)(\omega^2/c_s^2) + \omega^2 k_z^2}{\omega_g^2 - \omega^2} \quad k_z^2 = (\omega_g^2 - \omega^2) \frac{k_h^2}{\omega^2} - (\omega_a^2 - \omega^2)/c_s^2$$

$$\omega^2 = \frac{1}{2} \left[\omega_a^2 + c_s^2 k^2 \pm \sqrt{(\omega_a^2 + c_s^2 k^2)^2 - 4 c_s^2 k_h^2 \omega_g^2} \right]$$

Brunt-Väisälä frequency Schwarzschild convective instability criterion: $N^2 < 0$

$$\omega_g \equiv \frac{g}{c_s} \sqrt{\gamma - 1} \quad N_{BV}^2 \equiv \frac{g^2}{c_s^2} (\gamma - 1) + \frac{g}{T} \frac{dT}{dz} \quad \text{isothermal } \omega_g = N_{BV} \quad \gamma = 5/3 : \omega_g = 0.98 \omega_a$$

with $L^2 \equiv c_s^2 k_h^2$ and $\omega_g \approx \omega_a$

$$k_z^2 = \frac{1}{\omega^2 c_s^2} [(\omega_g^2 - \omega^2) k_h^2 c_s^2 - (\omega_a^2 - \omega^2) \omega^2] \approx \frac{1}{\omega^2 c_s^2} (\omega^2 - \omega_{g \approx a}) (\omega^2 - L^2)$$

diagnostic diagram

shaded: two ω roots per $(k_h, k_z) =$ propagative waves

$\omega > \omega_a$ and $\omega > L$: acoustic wave

$\omega < \omega_g$ and $\omega < L$: internal gravity wave

Cowling: p (pressure) or g (gravity)

blank: $k_z^2 < 0$: evanescent in z

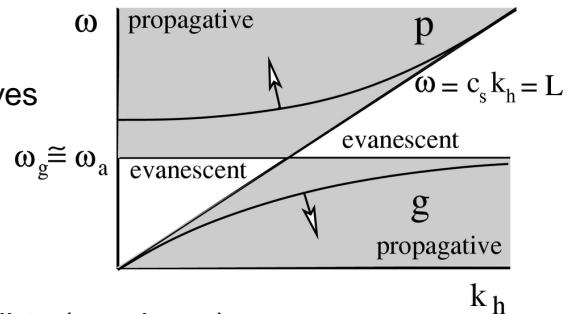
line $\omega^2 = c_s^2 k_h^2 = L^2$: horizontal "Lamb" waves

line $\omega^2 = (\omega_g/\omega_a)^2 c_s^2 k_h^2$: gravity-wave cutoff at small k_h (not shown)

upper curve: constant k_z , hyperbolic, $\omega^2 - \omega_a^2 \approx c_s^2 k_z^2$ for small k_h

lower curve: constant k_z , hyperbolic, $\omega_g^2 - \omega^2 \approx k_z^2/k_h^2$ for large k_h

arrows: increasing k_z



SLANTED AG WAVES IN ISOTHERMAL ATMOSPHERE 3

Stix Fig. 5.12

phase speed $\theta \equiv$ angle between \vec{k} and k_z

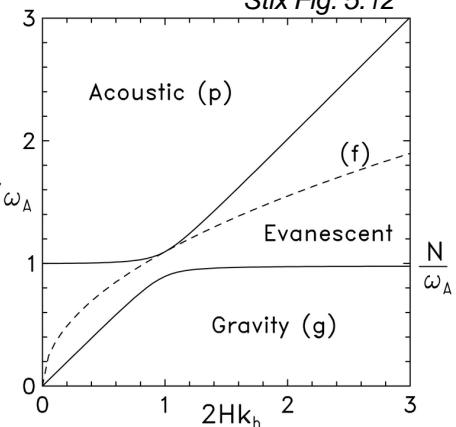
$$v_{\text{phase}}^2 = \frac{\omega^2}{k^2} = \frac{1}{2} \left[\frac{\omega_a^2}{k^2} + c_s^2 \pm \sqrt{\left(\frac{\omega_a^2}{k^2} + c_s^2 \right)^2 - 4 c_s^2 \sin^2 \theta \frac{\omega_g^2}{k^2}} \right]$$

$+ \sqrt{\dots} = p \text{ waves}, - \sqrt{\dots} = g \text{ waves}$

p waves: $v_{\text{phase}} \geq c_s$ $v_{\text{phase}} \downarrow$ for $\omega \uparrow$

g waves: $0 \leq v_{\text{phase}} \leq (\omega_g/\omega_a) c_s$ $v_{\text{phase}} \uparrow$ for $\theta \rightarrow \pi/2$

$v_{\text{phase}} = 0$ for $\theta = 0, \pi$



group velocity

$$v_{\text{group}}^{\text{hor}} = \frac{c_s^2 (\omega^2 - \omega_g^2) k_h}{\omega (2\omega^2 - \omega_a^2 - c_s^2 k^2)} \quad v_{\text{group}}^{\text{ver}} = \frac{c_s^2 \omega^2 k_z}{\omega (2\omega^2 - \omega_a^2 - c_s^2 k^2)}$$

pure p regime $\omega \gg \omega_a$

$$\omega^2 = c_s^2 k^2 \quad v_{\text{phase}} \approx c_s \quad v_{\text{group}}^{\text{hor,ver}} \approx (c_s^2/\omega) k_{h,v} \leq c_s \quad P_1, \rho_1, v_h, v_z \text{ in phase}$$

pure g regime $k_h^2 \gg \omega_g^2/c_s^2$

$$\text{dispersion: } \omega^2 = (k_h^2/k^2) \omega_g^2 = \sin^2 \theta \omega_g^2$$

non-vertical only ("blob" concept needs horizontal info)

$v_{\text{group}}^{\text{hor}} \sim k_h, v_{\text{group}}^{\text{ver}} \sim -k_z$ slanted upward waves transport energy downward

evanescent regime

k_z imaginary, k_h free \mathcal{P}, \mathcal{R} imaginary, \mathcal{Z} real: P_1 and \vec{v}_z 90 deg out of phase

non-radial fundamental mode (f) ~ ocean wind wave: $\omega = \sqrt{g k_h}$

WAVE TRAPPING

Stein & Leibacher 1974 ARA&A.. 12..407S

temperature sensitivities

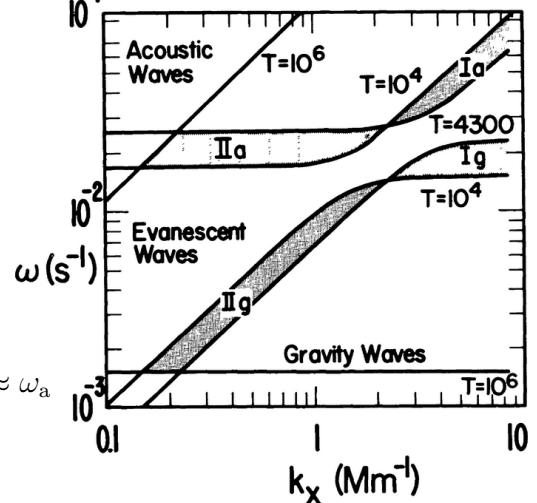
$$\text{sound speed: } c_s \equiv \sqrt{\gamma \frac{P_0}{\rho_0}} = \sqrt{\gamma \frac{RT}{\mu}} \sim \sqrt{T}$$

$$\text{acoustic cutoff frequency: } \omega = \omega_a \equiv \frac{\gamma g}{2 c_s} \sim 1/\sqrt{T}$$

$$\text{horizontal acoustic-wave line (Lamb): } \omega = c_s k_h$$

$$\text{gravity cutoff line: } \omega = (\omega_g/\omega_a) c_s k_h$$

$$\text{horizontal gravity-wave frequency: } \omega = N_{BV} \approx \omega_g \approx \omega_a$$



wave trapping

refraction at increasing wave speed (wavefront bending)

turn-around where waves become horizontal (node in k_z)

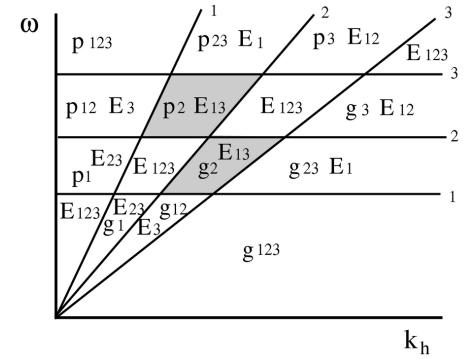
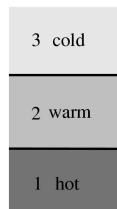
anti-node reflection at cutoff (infinite phase speed)

p -mode cavity walls: ω_a cutoff and Lamb-line refraction

g -mode cavity walls: gravity cutoff line and N_{BV} refraction

example: three temperature regimes

shaded: 2 = cavity p and g modes



SOLAR CAVITIES

Stix Fig. 5.13

p: refractive turnaround at Lamb line

$$\omega^2 = c_s^2 k_h^2 = c_s^2 \frac{l(l+1)}{r^2} \sim \frac{T l^2}{r^2}$$

lowest l modes probe deepest

g: refractive turnaround at Brunt-Väisälä N^2

no g waves in convection zone

interior g modes? lift whole envelope?

atmospheric g waves?

near solar center

Lamb reflection $\omega^2 \sim 1/r^2$

ω_a and $N_{BV} \sim g \sim r \Rightarrow N^2 \sim r^2$

only radial $p(l=0)$ modes reach center

five-minute oscillation ($\omega \approx 0.02$ Hz)

upper reflection = ω_a (here N_{AC})

interior reflection = Lamb turnaround
(≈ closed/open ends of my flute)

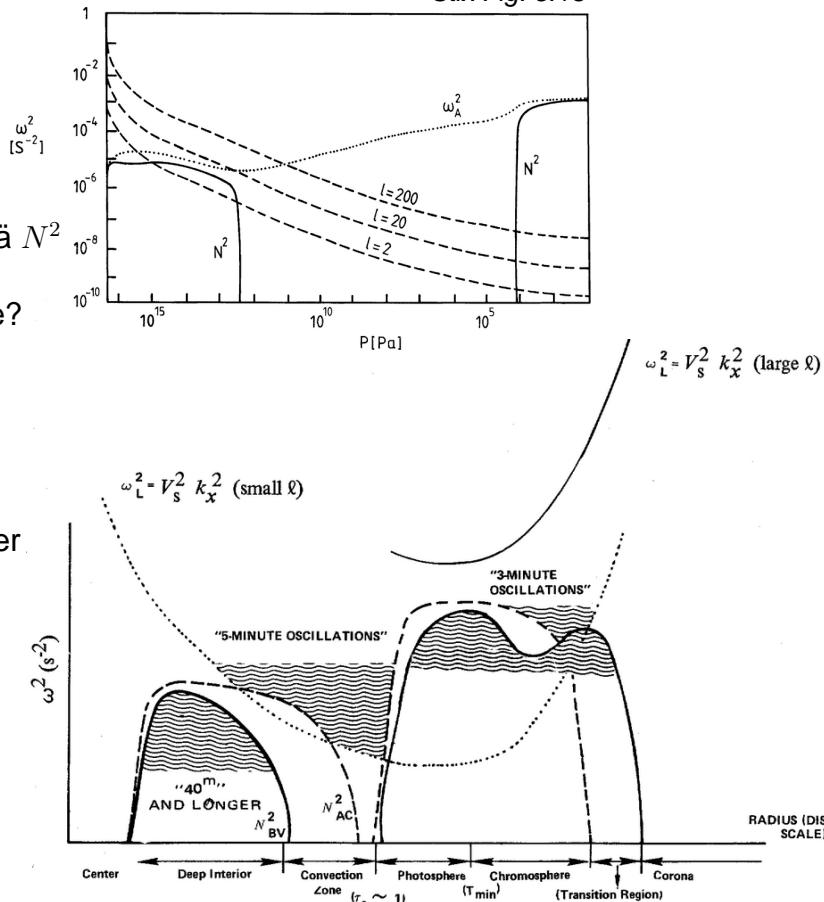
p modes evanescent where detected

three-minute oscillation ($\omega \approx 0.035$ Hz)

here: cavity from coronal T rise

but: T rise too warped

reflection/mode conversion at \vec{B}



POWER RIDGES $f-l$ (aka $k-\omega$) DIAGRAM

Stix section 5.2

assume constant temperature gradient with depth $d = -z$

$$\frac{dT}{dd} \approx C = \left(\frac{dT}{dd} \right)_{\text{ad}} \quad T(d) = \left(\frac{dT}{dd} \right)_{\text{ad}} d \quad -\frac{1}{T} \left(\frac{dT}{dd} \right)_{\text{ad}} = \frac{(\gamma - 1) g}{c_s^2} \quad c_s^2 \approx (\gamma - 1) g d$$

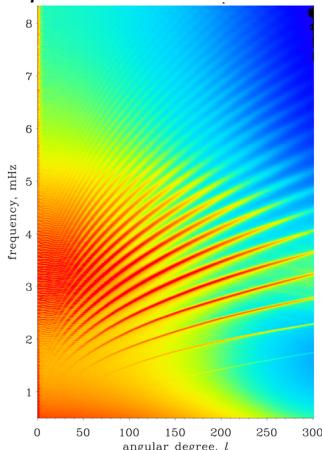
lower reflection occurs at depth $d = \delta$ where $\omega^2 = c_s^2 k_h^2$:

$$\delta = \frac{c_s^2}{(\gamma - 1) g} = \frac{\omega^2}{(\gamma - 1) g k_h^2}$$

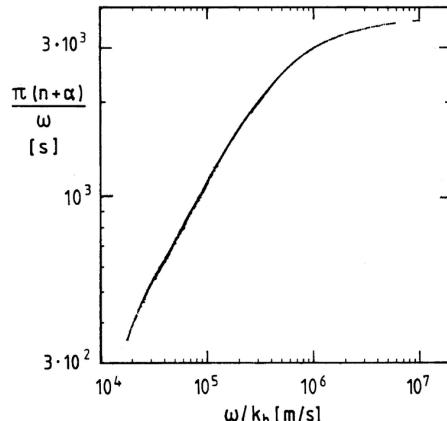
standing wave has $(n + 1/2) \pi = \int k_z dd = \omega \int_{d=\delta}^{d=0} \frac{1}{c_s} dd = \frac{2\omega \sqrt{\delta}}{\sqrt{(\gamma - 1) g}} = \frac{2\omega^2}{(\gamma - 1) g k_h}$

parabolas $\omega_n^2 = \frac{1}{2}\pi (n + 1/2) (\gamma - 1) g k_h$

<http://sohowww.nascom.nasa.gov>



Stix Fig. 5.16



Duvall's law with $\alpha = 1.58$

PDF SET/DIR FILE TABLE

present talk

talkstart	/home/strknd/rutten/rr/wrt/talkstart.pdf	sol-waves1	sol-waves10.pdf
sol-waves2	sol-waves3	sol-waves4	sol-waves5
sol-waves6	sol-waves7	sol-waves8	sol-waves9
sol-waves10	sol-waves11	sol-waves12	setfiletable